The Math and Science of Modeling: Scale Effect

WARNING: Fractions, formulas and basic math involved. If you don't like math and science, you won't like this.

I recently saw a rerun of a Myth Busters episode that exploded the myth that a postage stamp placed on at the tip of a blade on a helicopter's main rotor would take the vehicle out of the air. As part of the rigorous testing that Myth Busters is famous for, they used a 1/7 scale RC helicopter. The red haired girl showed what she presented as a 1/7 postage stamp. The stamp she showed was about 1/7 the *area* of a postage stamp, not a *1/7 scale* postage stamp. As a former math and science teacher, I was quite surprised with the error. What's the difference? Glad you asked!

Let's first look at exactly what scale means. Scales refer to a linear measurement, a one dimensional value. To find the length of a scale replica, we can use this handy formula:

Actual length x scale = scale length

Something 1 in long in reality, for example, will be 1/8 in long in 1/8 scale.

1in x 1/8 = 1/8in

A 1/72 scale 6 ft person, 72 in tall in the real world, would be 1in tall.

72in x 1/72= 1in

A more graphic example might help

4in real world line

Length of 1/16 replica 4in x 1/16 =4/16in = ¼in

1/16 replica of a 4in line

Let's try it backwards. The bottom number in a fraction is called the denominator. If you have the real world measure of the replica, you find the actual length of the real object by multiplying that replica's length by the denominator of the scale. So, if your 1/48 scale replica measures 1ft long, that corresponds to the real thing being 48ft long.

1ft x 48 = 48ft

This only works of the top number or numerator is 1. We'll leave what to do if the numerator isn't 1 for another day.

That's nice. What about the stamp? The stamp has two dimensions, length and width. As such, the stamp has *area*. Area is found by multiplying the length by the width.

You may recall this formula from a math class you had at some point in your K-12 education.

$A = l \times w$, or $A = lw$

Area is also expressed in square units. Square inches or in^2. It’s kind of like floor tiles. Each is a square.

For ease of explanation, let’s start with nice stamp that measures 1in x 2 in. The area of this stamp is easy to find.

$A = 1in \times 2in = 2$ sq in or $2\ in^2$

Now, let’s find the area of a ½ scale replica.

$l = 1$ in Scale $l = 1in \times 1/2 = ½$ in

w = 2 in Scale w = 2 in x ½ = 1 in

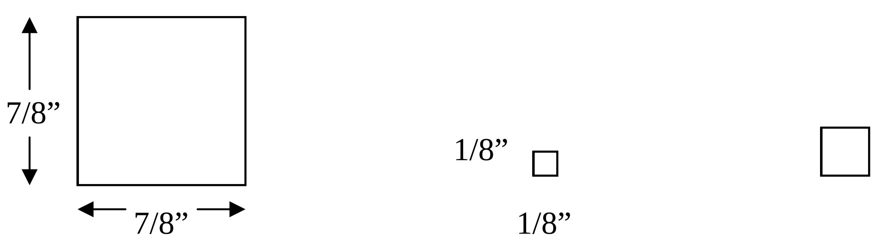
Scale A = Scale l x Scale w = ½ in x 1 in = ½ in^2

That’s ¼ the original area. (2 x ¼ = ½) If the scale is 1/8, that will be 1/8 x 1/8 or 1/64 the original area.

So, the correct size of a 1/7 scale postage stamp would be 1/7 of the length times 1/7 of the width. That’s 1/49 the original area.

A = 1/7 x 1/7 = 1/49 the original area.

A more graphic example. For simplicity, each edge measures 7/8”



Original Stamp

1/7 Replica
(1/49 original area)

Myth Busters “1/7” Replica
(Approx. 1/7 original area)

As you can see, big difference!

That’s nice. Who cares? Well, what if this related to adding another level to the accuracy of your models? Consider this question. How much should that 1/35 scale tank weigh?

Time for two new concepts! Volume and density.

Volume is a measure of how much *3 Dimensional Space* an object occupies. Three dimensions: length, width and height. You may recall this formula.

V= l x w x h or V=lwh

That's three dimensions multiplied together. Volume is measured in cubic units. Cubic inches (come on, car guys!) or in^3

For example, a ½ scale replica of 1 in^3 would be as follows:

Scale *l* = 1 in x ½ = ½ in   Scale w = 1 in x ½ = ½ in   Scale h = 1 in x ½ = ½ in

Scale V = ½ in x ½ in x ½ in = 1/8"   (2 x 2 x 2 =8)

If the scale is ¼, that would be 1/64 the original volume. (4 x 4 x 4 = 64) If the scale is 1/35, that would be 1/42,875 the original volume. (35 x 35 x 35 = 42,875)

That's nice. What about the weight of a 1/35 scale tank?

The missing piece here is that other new concept: density. Density is the measure of an object's mass per unit of volume. Mass is the amount of matter present. An object's weight is the force that gravity exerts on that mass. Not the same thing (Imagine pushing a car, mass, versus lifting a car, weight) but for our discussion, we will allow them to be the same. And better yet, because we are momentarily making mass and weight the same, we don't need any new formulas!!!

To strengthen understanding of the concept of density, consider a gallon jar of feathers and a gallon jar of lead. They both take up the same amount of space but, the gallon of feathers weighs a lot less. So, feathers are less dense that lead. Similarly, a gallon of water would weigh much more than a gallon of air. Water is much denser than lead.

So, let's say a real tank weighs in at 60 tons. That translates to 120,000 lbs. (60 tons x 2,000 lb/ton = 120,000 pounds) Let's also assume for the moment that the density of the real tank is equal to the density of that tank in a 1/35 scale universe. So, if it has 1/42,875 the volume, it should also have 1/42,875 the mass. That means this:

120,000 lb x 1/42,875 = 2.8 lb = 44.8 oz (1 lb = 16 oz)

So, your 1/35 replica of a 60 ton tank should weight about as much as 45 pieces of mail with $.39 stamps on them. How about the same weight as 11 McDonald's Quarter Pounders? (Remember, that's before cooking and no buns or anything else. ¼ lb = 4 oz)

What about a 1/6 scale male figure? Let's let the real guy be 6 ft or 72 in tall and weigh 185 lb or 2960 oz. Our scale is 1/6 so that means there would be 1/216 the volume. (6 x 6 x 6 = 216)

2960 oz x 1/216 = 13.7 oz. That's just the guy. No clothing or equipment

That's almost 3 ½ Quarter Pounders before cooking and without buns or anything else. That's about 14 pieces of $.39 stamped mail.

How about an aircraft that weight 2 tons at take off? Consider a 1/72 model.

2 tons = 4000 lbs = 64,000 oz   72 x 72 x 72 = 373,242

64,000 oz x 1/373,242= 0.17 oz   Must be a light plane!

So there ya go! Now you can calculate the scale weight of your models and make adjustments as necessary. How knows? If this catches on, judges may start carrying analytic balances along with their mirrors and flashlights! Or not!